Particularities of Hybrid Electric Vehicles' Dynamics

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Abstract – In this paper the main particularities of hybrid electric vehicle's dynamics are highlighted, depending on the vehicle's configuration. The main elements which define the configuration architecture are presented as well as the mathematical model used. Mathematical descriptions are presented here, both for the mechanical and electric components. We also present advantages and disadvantages of the basic configuration solution.

Index Terms – Automotive, dynamics modeling, hybrid electric vehicle, hybrid power train, vehicle dynamics

I. INTRODUCTION

New solutions were developed in automotive engineering due to the fact that the number of vehicles is fast increasing, new and more drastic pollution emissions standards are imposed, the necessity to diminish the consumption of oil...all these led to the development and the emerge of new energetic solutions in the automotive industry; speaking of the solution which is the forefront of the industry we look at the hybrid electric vehicles which combine electromechanical and electro-chemical systems of great complexity [4].

Most of electric hybrids are powered by thermal and electric motors, with various solutions of power train configuration: series, parallel, sequential and power (torque) split. The thermal engines are used to move at high speeds, thus high engine revs where the efficiency has high values and the emissions are reduced. When starting the vehicle or when moving it at low speeds, electric motors are used because these motors develop high torque value at low revolutions values, and because the thermal engine is most pollutant at these revolution values and highly instable.

In Fig. 1, the series configuration of a hybrid electric vehicle is presented, where there is no mechanical connection between the thermal engine and the wheels; in the figure we used the following notations: F - fuel; TE - thermal engine; EG - electric generator; EM - electric motor; GB - gear box; BP - battery pack; EI - electronic interface. The thermal engine must ensure the energy conversion... that of the fuel's into mechanical energy. Further on, the electric generator transforms the mechanical energy into electricity, thus allowing the battery to be recharged so it can power up the electric motor which ensures the vehicle's movement. As we can see, there are several energy conversions and that is why the efficiency of a system like this one is very low [3].

The most simple series configuration example is represented by the pure electric vehicle, which obviously has the advantage that has no polluting emissions of any kind. So, a clear advantage consists of the fact that it ensures low polluting emissions because in order to move the vehicle, we act on it electrically.



Fig. 1 Series configuration

Another advantage consists in the fact that the electric and internal combustion engines can be individually mounted in any convenient positions; this ensures the advantage of distributing the weigh evenly on the axels or as we consider to be fit.

The name of series configuration comes from the fact that the power flow of the two power sources are positioned in series. The mathematical expression described in the first relation (1) represents the differential equation which describes the hybrid vehicle's dynamic behavior with series topology. It is a result following the application of the d'Alambert principle, similar as the classical vehicle [1].

$$\frac{\mathrm{d}v_s}{\mathrm{d}t} = \frac{1}{r_r m_a} \left(M_b + M_r + M_{EM} i_t \right) \tag{1}$$

where: v_s – the speed with which the vehicle is moving (the *s* letter comes from the word series), r_r – rolling radius, m_a – vehicle's mass, M_b – braking torque, M_r – required torque needed to overcome the resisting forces when moving forward, M_{EM} – the torque generated by the electric motor, i_t – transmission ratio.

We can see in Fig. 2 the sketch for parallel configuration of an electric hybrid vehicle, on which we can see the fact that there is a mechanical connection between the internal combustion engine and the electric one. The name comes from the fact that the power flows if the two sources are parallel. The solution has a better efficiency than the series configuration because there are fewer energy conversions [3].

We can see from this sketch that the vehicle can move in four distinct ways: relying only on it's electric power; relying only in it's internal combustion engine and at the same time the combustion engine is charging the battery that powers the electric motor; and at the same time the two engines can power up the car at the same time.



Fig. 2 Parallel configuration

This is where it derived the version of dual transmission functioning method, which exists on most hybrid electric vehicles.

The second mathematical expression represents the differential equation which describes the parallel hybrid vehicle's dynamics. It also results from applying the d'Alambert principle.

$$\frac{\mathrm{d}v_p}{\mathrm{d}t} = \frac{1}{r_r m_a} \Big[M_b + M_r + M_{EM1} i_{t_1} + (M_{TE} + M_{EM2}) i_{t_1} i_{t_2} \Big]$$
(2)

The used notations are similar with the previous ones, M_{TE} being the torque developed by the thermal engine.

Fig. 3 presents a sequential configuration of a hybrid electric vehicle. It is a combination of both parallel and series configurations. Mounting a clutch mechanism C on the lower branch from the parallel configuration, it is possible to change between parallel into series configuration.

Engaging or disengaging the clutch, thus changing from one sequence onto another, is being done by the on board computer in such a way that the power losses are minimized. From this reason this solution it is named combined configuration; thus we can say that the vehicle's dynamics is described either by the firs mathematical relation (1) either by the second one (2) depending on the sequence that the vehicle finds its self.



Fig. 3 Sequential configuration

Fig. 4 presents the configuration with "power (torque) split", on which a planetary mechanism PM is present which connects the internal combustion engine with two electric motors. The electric motor EM1 is a traction engine and is connected to the ring of the mechanism, as you can see in Fig. 4. The electric motor EM2 is a generator and is connected to the sun. Finally, the internal combustion engine is connected to the planet carrier thus offering the possibility of turning it off and moving the vehicle based only on the power generated by the EM1 [2, 3].

II. MATHEMATICAL MODEL

Most of the cases the vehicle functions when there is a power flow coming from the internal combustion engine, like the parallel configuration, but also from the electric motor EM1, as the case for series configuration. The proportion between these two power flows depends on the revolution's speed. The name of this solution comes from the fact that the planetary geared mechanism ensures a power (torque) split; at the planetary transmission the following notations were made: C - carrier; S-sun; R - ring.



Fig. 4 "Power (torque) split" configuration

Based on Fig. 5, the mechanism's dynamics and kinematics relations are established.

So, the mechanism's kinematics is described by the third relation (3):

$$\omega_R r_R + \omega_S r_S = \omega_C (r_R + r_S) \tag{3}$$



Fig. 5. Mechanism kinematics

And its dynamics is described by the differential equations (4):

$$J_{R} \frac{\mathrm{d}\omega_{R}}{\mathrm{d}t} = Fr_{R} - M_{R}; J_{C} \frac{\mathrm{d}\omega_{C}}{\mathrm{d}t} = M_{C} - Fr_{R} - Fr_{S}; J_{S} \frac{\mathrm{d}\omega_{S}}{\mathrm{d}t} = Fr_{S} - M_{S}$$
(4)

Based on Fig. 5b we also have the following:

$$M_{c} - M_{R} - M_{s} = 0; \quad M_{c}\omega_{c} - M_{R}\omega_{R} - M_{s}\omega_{s} = 0 \quad (5)$$

From (3) and (5) we get:

$$M_{R} = \frac{\omega_{C} - \omega_{S}}{\omega_{R} - \omega_{S}} M_{C}; M_{S} = \frac{\omega_{R} - \omega_{C}}{\omega_{R} - \omega_{S}} M_{C}$$
(6)

Relations (6) show how the mechanism splits the torque. It is from here that the name of "torque split configuration". These expressions give us the torque values on the sun gear and on the ring gear considering the torque of the planet carrier; according to Fig. 5a, the expressions described in relations number (6) offers the torque values for the electric motors depending on the torque provided by the internal combustion engine.

In order to establish the vehicle's dynamic equations, we use the sketch from Fig. 6 (MG: motor-generator) where, just as the case of Fig. 5b, we can see the components, the forces, torques and specific angular rotational speeds [3, 5].

So the moving law of the vehicle is being deducted from the relations described in the mathematical expressions number (7):

$$\left(J_{MG2} + \frac{m_a r_r^2}{i_t^2}\right) \frac{\mathrm{d}\omega_R}{\mathrm{d}t} = M_R + M_{MG2} - \frac{1}{i_t} (M_b + M_r); \quad \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{r_r}{i_t} \frac{\mathrm{d}\omega_R}{\mathrm{d}t}$$
(7)

...as well as from the mathematical expressions number (4); we finally achieve equation number (8), which describes the vehicle's dynamics:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{r_r}{J_R i_t^2 + J_{MG2} i_t^2 + m_a r_r^2} \left(F r_R i_r + M_{MG2} i_r - M_b - M_r \right)$$
(8)

Just as the classic dynamic equation [1], in the given relations, inertial torques intervene, constructional elements of the vehicle, external torques; in this case inner torques and forces intervene, for example those from the electric motors and the inner F forces of the planetary gear mechanism.



Fig. 6 Explanatory sketch

For a hybrid electric vehicle, on the given expressions, relation number (9) is added which gives us the time variation of battery charging degree, know in the technical literature as SOC – State of Charge [5].

$$\frac{dSOC}{dt} = -\frac{V_b - \sqrt{V_b^2 - 4(M_{MG1}\omega_{MG1}\eta_{MG1}^k \eta_{MG1}^k + M_{MG2}\omega_{MG2}\eta_{MG2}^k \eta_{c2}^k)}{2R_b Q_{m}}$$
(9)

Fig. 7 presents the specific battery parameters, which are found in the mathematical expression number (9); besides that we have battery power P_b and maximum battery capacity Q_m .



Fig. 7 Battery electric circuit

The mathematical expression (9) represents the power supplied to the MG:

$$P_{MG} = M_{MG} \omega_{MG} \eta_{MG}^k \tag{10}$$

where: M_{MG} and ω_{MG} represent the torque and respectively the rotational speed. If the velocity and torque of the MG have the same signs (i.e., either positive or both negative), the power is positive, which means that the motor is consuming energy. Similarly, if the signs for velocity and torque differ (i.e., one positive, the other one negative), the MG is generating energy. In the mathematical expression (10), *k* is the sign for the power flow direction. When MG is consuming energy, *k*=-1 and the power flows inward, from the battery to the MG. When the MG is generating energy, *k*=1 and the power flows outward, from the MG to the battery. The efficiency η_{MG} accounts for the energy lost from both the MG (MG1 and MG2) and other accessories, including the power converter and controller.

The mathematical expression (9), η_c represents the power converter efficiency, and k is the sign of the power flow direction as explained above. When the battery is discharged, k=-1 and the power flows away from the battery. When the battery is charged, k=1 and the power flows to the battery.

There are several specialized software programs which deal with the hybrid electric vehicles: ADVISOR, Matlab, Dymola, and AmeSim etc. As an example, the Matlab sketch presented in Fig. 8 shows the electric hybrids' dynamics, thus further detailing the electric subsystem.



Fig. 8 Matlab sketch - electric hybrids' dynamics

Further on, there are some results which were achieved using the sketches from Fig. 8 and Fig. 9, as well as data regarding the vehicle, which was a Toyota Prius which had installed a THS (Toyota Hybrid System) thus splitting the generated torque.



Fig. 9 Matlab sketch



Fig. 10 Characteristic parameters of the high voltage battery pack

So, Fig. 10 presents some characteristic parameters for the high voltage battery pack.

Fig. 11 presents the angular speeds for the planetary transmission's components.



components

Fig. 12 presents the specific torque values for the planetary components and Fig. 12d checks the equality described by the mathematical relation no. (5).



Fig. 12 Specific torque values for the planetary components

Fig. 13 presents specific parameters for the vehicle's internal combustion engine.

Finally we can see in Fig. 14 specific parameters for the entire vehicle. The scheme from Fig. 8 allows us to establish the behavior for other functional parameters of the internal combustion engine, of the electric motors MG1 and MG2, of the electric converter, of the vehicle and so on...



Fig. 13 Specific parameters for the vehicle's internal combustion engine



Fig. 14 Specific parameters for the entire vehicle

In a similar manner the mathematical model can be established for any hybrid electric vehicle configuration, let's say in the case of a transmission with two working ways (dual-mode power split power train).

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